Modeling Data Durability and Availability

Serkay Ölmez
serkay.olmez@seagate.com

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Acknowledgements:
Ian Davies, Mike Barrell, John Bent, and Iman Anvari

Outline

- Modeling Failures
  - Weibull Distribution
- Erasure Coding(EC)
  - General formalism, RAID 5& 6
- Hard Errors (UREs)
  - Modeling durability with UREs
- Distributed Parity
  - Improving data durability with ADAPT
- Multi-Layer EC
  - Improving durability with two layers of EC
- Availability
  - Modeling data availability
- Appendix
  - MACH2 and ReMan

Goals & Summary

Goals

- Provide a quick review of available models to compute data durability,
- Present an accurate and rigorous model,
- Establish a common language to compute these metrics.

Summary

A quick survey on literature

- The durability and availability of data can be predicted accurately with Markov Chains:
  - Based on rigorous math, and verified with Monte Carlo simulations.
  - Supports Distributed Parity, ReMan, UREs, Weibull failure modes, and multi-layer EC.
  - Developed in collaboration with the CORTX architects & sales team.
- Advanced features, such as Online ReMan, can be modeled too:
  - We continue to work on modeling latest and greatest CORTX features.

Modeling component failures

We will assume that individual component failures can be described by a Weibull distribution [10]. The failure probability density, cumulative failure distributions and the hazard rate (failure rate) are defined as follows:

$$f_{\alpha,\beta}(t) = \frac{\beta}{\alpha}\left(\frac{t}{\alpha}\right)^{\beta-1} e^{-\left(\frac{t}{\alpha}\right)^{\beta}}, \tag{1}$$

$$F_{\alpha,\beta}(t) = \int_0^t d\tau f_{\alpha,\beta}(\tau) = 1 - e^{-\left(\frac{t}{\alpha}\right)^{\beta}}, \tag{2}$$

$$h_{\alpha,\beta}(t) = \frac{f_{\alpha,\beta}(t)}{1 - F_{\alpha,\beta}(t)} = \frac{\beta}{\alpha}\left(\frac{t}{\alpha}\right)^{\beta-1}. \tag{3}$$

The actual values of α and β vary from product to product: β is expected to be between 1 and 2.

$\beta = 1$ gives the exponential distribution, which has completely random head failure times with a fixed failure(hazard) rate: $h_{\alpha,\beta}(t) = \frac{1}{\alpha} \equiv \lambda$. This simplifies Eqs. (1) ,(2) and (3) to:

$$f_\lambda(t) = \lambda e^{-\lambda t}, \quad F_\lambda(t) = 1 - e^{-\lambda t}, \text{ and } \quad \mathrm{h}_\lambda(\mathrm{t}) = \lambda. \tag{4}$$

Visualizing failure distributions

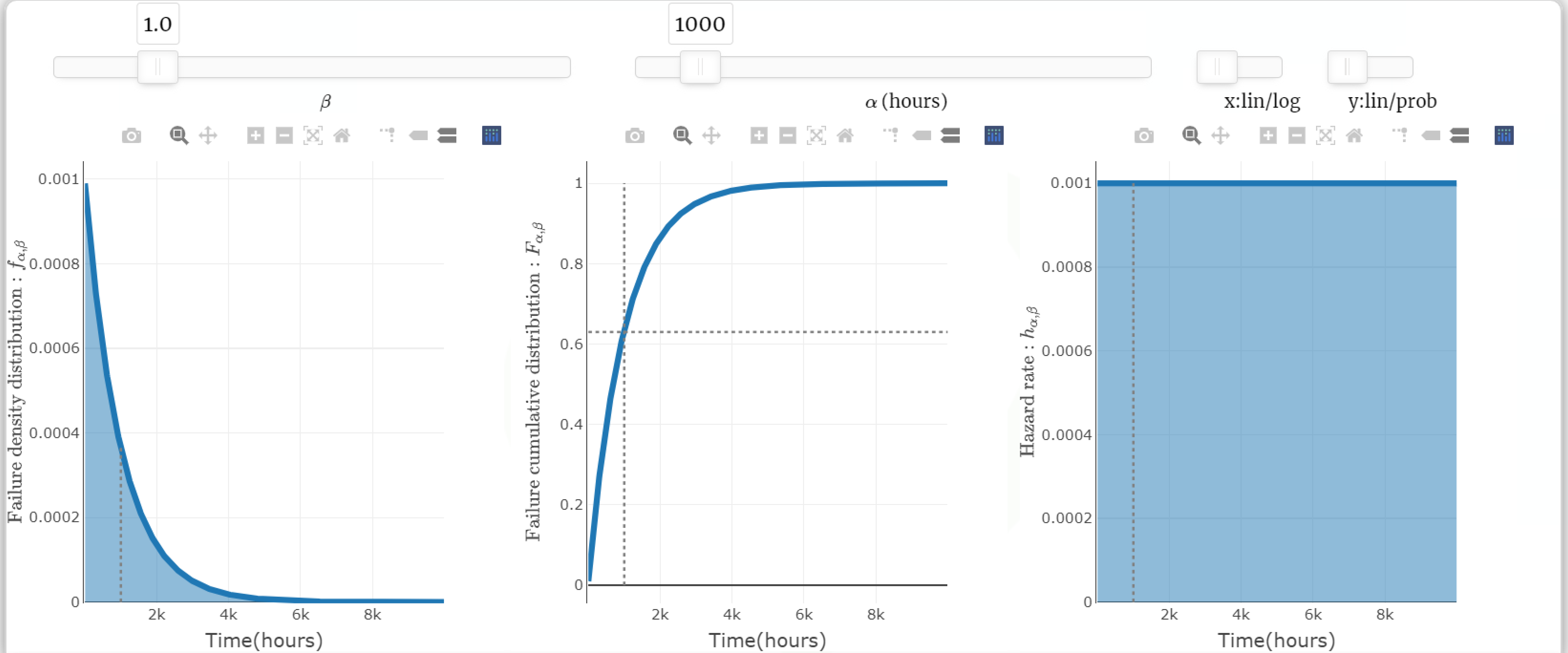


Figure 1: An interactive plot of Weibull distributions defined in Eqs.(1) ,(2) and (3) . Use the toggles for linear/log scales. ... [+]

- It is important to understand how the storage devices fail. Weibull is typically a good fit.
- No matter how reliable individual devices are, failures are inevitable.

Erasure Coding

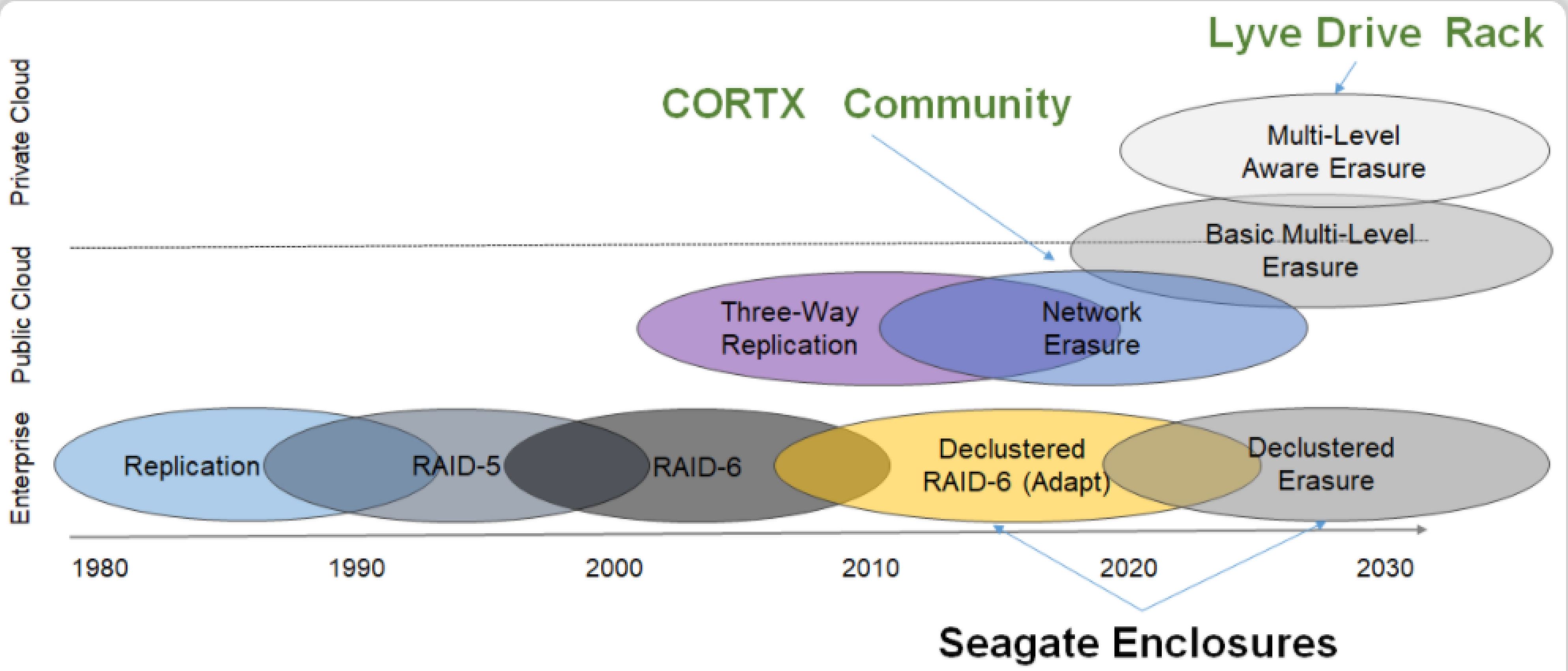


Figure 2: A brief timeline of the erasure coding. ... [+] Credit:John Bent

- The simplest way of creating redundancy is replication, but this has a very poor capacity efficiency.
- RAID 5 and Raid 6 introduce parities to protect data against device failures.
- Seagate enclosures supports declustered RAID6, which can be coupled with a top layer EC in CORTX to get the highest durability with best capacity efficiency.

Creating Redundancy

- EC adds fault-tolerance by creating redundant data pieces.
- Data is distributed across different storage media.

- Simplest example: compute and store the parity data.
  - Given two bytes of data: $B_1 = 01001001$ and $B_2 = 11011010$
  - The parity data: $P = B_1 \oplus B_2 = 10010011$
- Assume the drive that stored B_1 fails:
  - Compute $P \oplus B_2$, which is equal to B_1.
- The data on the failed drive can be reconstructed.
  - Fault tolerant to one drive failure.

- c pieces of redundancy data:
  - Parities computed using *Reed-Solomon algorithm.*
  - Fault tolerant to c simultaneous drive failures.
- **Main Question:** What is the probability of having $c + 1$ simultaneous failures?

RAID 5 & 6 (with URE)

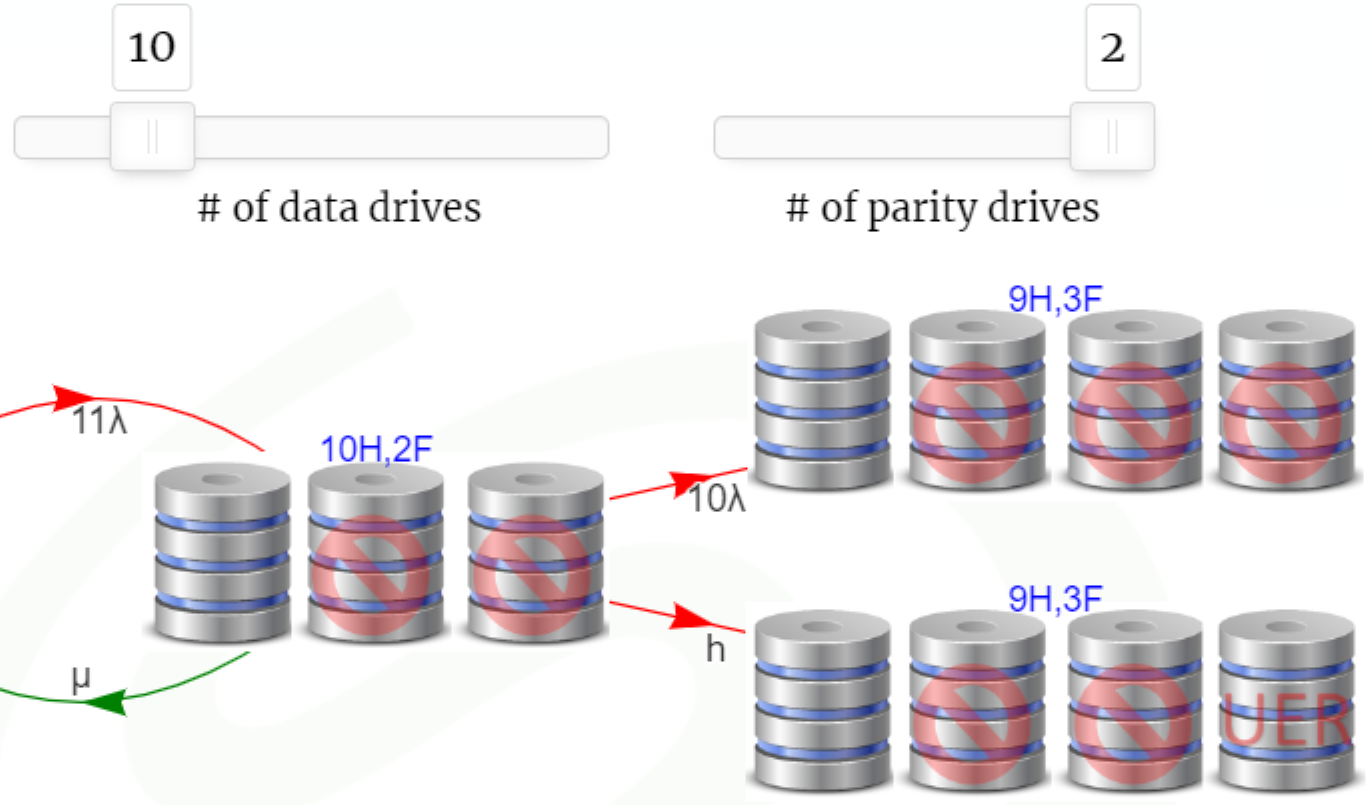


Figure 3: A Symbolic representation of the RAID6 including URE failures. Use the input sliders to change the number of drives or the redundancy. ... [+]

- The red arrows represent drive failures. Rate is scaled with the total number of drives:
  - λ is the failure rate per drive, and $n\lambda$ is the total failure rate for n drives.
  - The arrow denoted with h represents the data loss due to UREs- to be discussed in more detail later.
- The green arrows represent repairs. Failed drives are replaced, and data is rebuilt.
  - The repair time, $1/\mu$, depends on capacity and DR. It may be as long as several days.
- Data is lost when the system moves to the right-most state.
  - **Key metric: Mean Time to Data Loss (MTTDL)**: it is a function of n, c, λ, μ, drive capacity and UER.

Durability with minimal math

Here we review a few different approaches in the industry to compute data durability.

Press down arrow to navigate this sections.

The simplest model

Consider the following set up:

(n) drives 20 , capacity(C): 16 TB, redundancy(c): 3 , recovery speed (S): 50 MB/s, and AFR: 1 %.

- At this recovery rate, the recovery time from a drive failure is: (16 TB)/(50MB/s) = 3.7 days.
- The probability of losing a single drive in 3.7 days is: 1 % * (3.7 days/365)= 0.01%
- The system will lose data when there are 4 failures in 3.7 days, which has the probability: $(0.01\%)^4$.
- Data durability= 1- Probability of 4 failure(s) in 3.7 days=1- $(0.01\%)^4$ = 15 nines.

Number of nines is defined as the instances of leading 9's in reliability: 0.998 has 2 nines, 0.9991 has 3 nines.

$$\text{number of nines} = \text{Floor}\left(-log10(1 - \text{Durability})\right) \tag{5}$$

- This model is very simple, but... it is wrong!
- The metric calculated is **not** the data durability for an 3-redundant EC, it is for 1+3 [original+3 mirror(s)].
  - Note that the total number of drives, 20, did not even enter the equations!
  - There are binomial(20,4)=4845 combinations to choose 4 failure(s) out of 20 drives.
- This is not even the durability for over a year, it is just over 3.7 days. There are 99 such frames in a year.
- With the binomial coefficients and the # frames/year included, this model over-reports durability by at least 5 nines.
- This model also ignores UREs.

The BackBlaze Model

This is a model introduced by BACKBLAZE [11], [12].

(n) drives 20, capacity(C): 16 TB, redundancy(c): 3, recovery speed (S): 28.5 MB/s, and AFR: 0.4 %.

- The recovery time from a drive failure is: $T \equiv \frac{C}{S}$ = 6.5 days.
- The AFR value can be converted to the failure rate as: $\lambda = -\frac{\ln(1-\text{AFR})}{\text{year}} \simeq \frac{\text{AFR}}{\text{year}}$ = 0.004/year.
- The probability of a given drive to fail in 6.5 days is: $p_f \equiv \lambda T$ = 0.01%.
- The system will **not** lose data when there are at most 3 failures in 6.5 days.
  - The probability of not losing data in 6.5 days is: $p_{\text{no-loss}} = \sum_{i=0}^{c} \binom{n}{i} (p_f)^i (1-p_f)^{n-i}$ =13 nines.
- There are $N_F = \frac{365}{T}$ =56 such frames in a year.
  - The probability of not losing data in a year is: $(p_{\text{no-loss}})^{N_F}$ =11 nines.

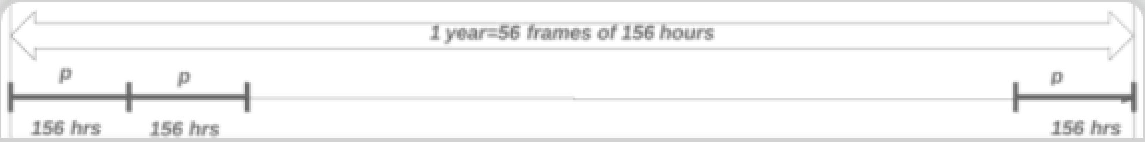


Figure 4: An illustration of the time frames.

- This model is still simple, but... it has a few issues:
  - The segmentation of a year into 56 chunks of 6.5 days implies that data is lost only when 4 failures happen in a given frame. The cases of 4 failures within in 6.5 days but spanning two subsequent frames are missed.
  - It is implicitly assumed that every frame starts with a 3-redundant system. In reality, there may be up to 3 ongoing repairs exiting the previous frame, and a single drive failure early in the next frame will cause data loss.
- Ignoring UREs, BackBlaze model works reasonably well in the low AFR limit (up to a factor of ~ 2).
- Including UREs will reduce the data durability by ~2 nines.

The most intuitive model

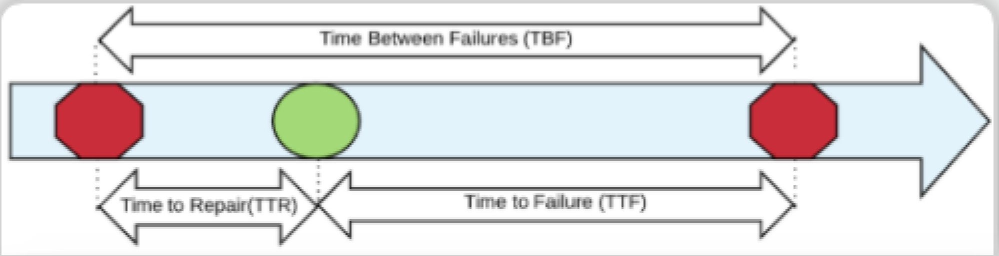


Figure 5: An illustration of the time scales.

- Figure plotref_avaldef_NTT shows the time scales in a repairable system.
- A system of n drives has the Mean Time To Failure: $\mathrm{MTTF} = 1/(\mathrm{n}\lambda)$.
- A failed drive is replaced, and rebuilt. Mean Time To Repair: $\mathrm{MTTR} = 1/\mu$.
- Fraction of the time spent for repair: $\frac{\mathrm{MTTR}}{\mathrm{MTTF+MTTR}}$

- Fraction of the time spent for repair: $\frac{\mathrm{MTTR}}{\mathrm{MTTF+MTTR}} \simeq \frac{\mathrm{MTTR}}{\mathrm{MTTF}} = \frac{n\lambda}{\mu}$ $(\mathrm{MTTF} \gg \mathrm{MTTR})$.
- There are $N-1$ drives left running. The rate of failure: $(n-1)\lambda$
- Multiply this rate with the fraction of time in recovery: $\frac{n\lambda}{\mu}(n-1)\lambda = \frac{n(n-1)\lambda^2}{\mu}$
  - This is the rate of data loss. Inverting the expression: $\mathrm{MTTDL}_1 = \frac{\mu}{n(n-1)\lambda^2}$
- When there are two parity drives, we can iterate to get: $\mathrm{MTTDL}_2 = \frac{\mu^2}{n(n-1)(n-2)\lambda^3}$

- For c parity drives, we can recursively iterate to get: $\mathrm{MTTDL}_c = \left[\frac{\mu}{\lambda}\right]^c \frac{(n-c-1)!}{\lambda n!}$
- $c=1$ is a RAID5, and $c=2$ is a RAID6 setup. $c \geq 3$ cases are refereed to as Erasure Coding in general.
- This model is very intuitive, and works great, but... it is not clear how to include UREs.

Durability with rigorous math

- Figure shows the Markov chain for a system of n drives with one redundancy.
- Markov chain represents state transitions and state probabilities $p_j(t)$.
- The change in $p_j(t)$ is dictated by incoming and outgoing arrows.

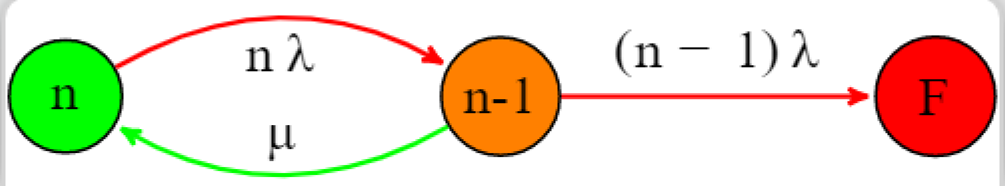


Figure 6: Markov Chain with one redundancy.

Show the details

- The state transitions are described by a set of coupled DEs.
- They can be solved by Laplace transforms.
- The equations are solved for the data loss state.
- The reliability: $\mathrm{R}(t) = 1 - p_{\mathrm{F}}(t) \sim e^{-t/\mathrm{MTTDL}_1}$
- $\mathrm{MTTDL}_1 = \frac{1}{n(n-1)\lambda}\frac{\mu}{\lambda}$.

$$\begin{aligned} \dot{p}_n + n\lambda p_n - \mu p_{n-1} &= 0 \\ \dot{p}_{n-1} + (n-1)\lambda p_{n-1} + \mu p_{n-1} - n\lambda p_n &= 0 \\ \dot{p}_{\mathrm{F}} - (n-1)\lambda p_{n-1} &= 0 \end{aligned} \tag{15}$$

- Markov chain analysis is needed to address complicated cases:
  - Declustered Parity (ADAPT),
  - Re-Manufacturing in the field (ReMan),
  - Generic Weibull distribution for drive failures ($\beta \neq 1$),
  - Latent sector errors, i.e., hard errors (UREs).
- Most of the items above will be addressed in this presentation.

Monte Carlo Simulation

Show the simulation script (JSL)

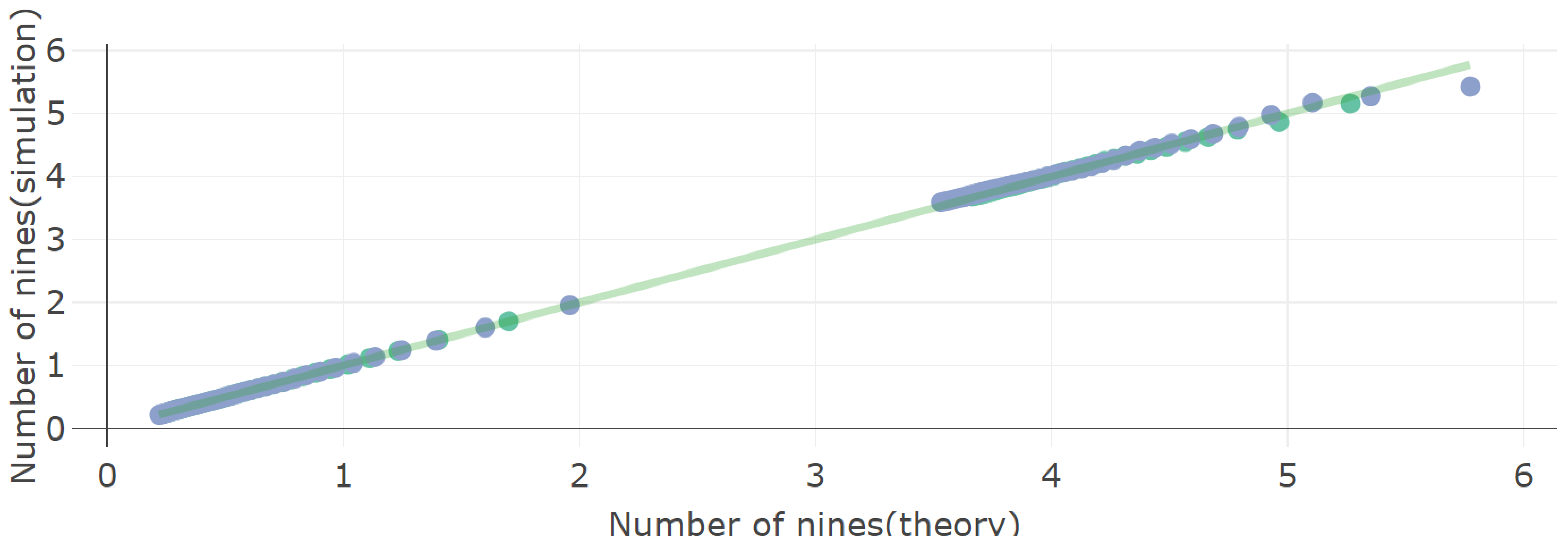


Figure 8: The comparison of theory vs Monte Carlo results with various input parameters. Number of nines is defined in Eq. (5) where flooring is omitted. ... [+]

Silent data corruption (URE)

- UREs may arise due to thermal decays of bits. They are discovered only when data [is attempted to be] read.
- It is defined as the probability of a corrupted sector **per bits** read. A typical value is 10^{-15}.
- With tens of TBs data, observing at least one corrupted sector is very likely [13].

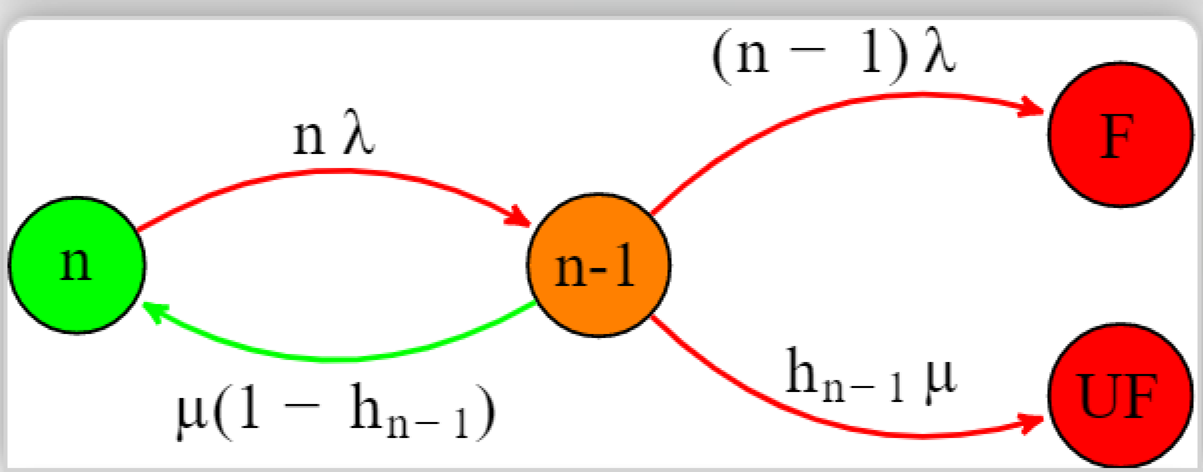


Figure 9: Markov chain with one redundancy with UREs.

- Figure on the left includes failures due to UREs.
- $$S = \begin{bmatrix} s+n\lambda & -\mu(1-h_{n-1}) & 0 \\ -n\lambda & s+(n-1)\lambda+\mu & 0 \\ 0 & -(n-1)\lambda - h_{n-1}\mu & s \end{bmatrix}$$
- h_{n-1} is the probability of observing UER(s) in critical repairs:

$$h_{n-1} \equiv 1-(1-\mathrm{UER})^{\mathrm{N_{bits}}} \simeq 1-\mathrm{e}^{-\mathrm{UER}\times\mathrm{N_{bits}}} = 1-\mathrm{e}^{-\mathrm{UER}\times(\mathrm{n}-1)\times\mathrm{C_{bits}}} \quad (16)$$

- Finding the poles of the determinant we get: $\frac{1}{\mathrm{MTTDL}} = \frac{\mu}{n\lambda[(n-1)\lambda]} + \frac{1}{1/(n\lambda)} h_{n-1}$
  - This is an harmonic sum of MTTDLs for Drive failure mode and UER failure mode.
  - The analysis can be extended to a generic c redundancy: $\frac{1}{\mathrm{MTTDL}_c} = \frac{1}{\mathrm{MTTDL}_{c,\mathrm{DF}}} + \frac{h_{n-1}}{\mathrm{MTTDL}_{c-1,\mathrm{DF}}}$
- For large data $[UER \times (n-1) \times C_{\mathrm{bits}} \gg 1]$, h_{n-1} is significant, and the second term dominates the durability.
  - Ex: $\mathrm{UER} = 10^{-15}$, $n = 20$ and $C = 10\mathrm{TB} \to h_{n-1} = 1-(1-10^{-15})^{19*8*10^{13}} \simeq 1-e^{-1.52} = 0.78$.
- Sector level data durability of an EC with c parities is at the order of drive level durability with $c-1$ parities.

Distributed Parity (ADAPT)

- Distributed raid (dRAID) uses all the drives in the pool to store data and parities.
- Rebuild is done by reading from all drives in the pool in parallel.
- ADAPT prioritizes the repair of critically damaged stripes[9]. This is the main reason for reli gain.

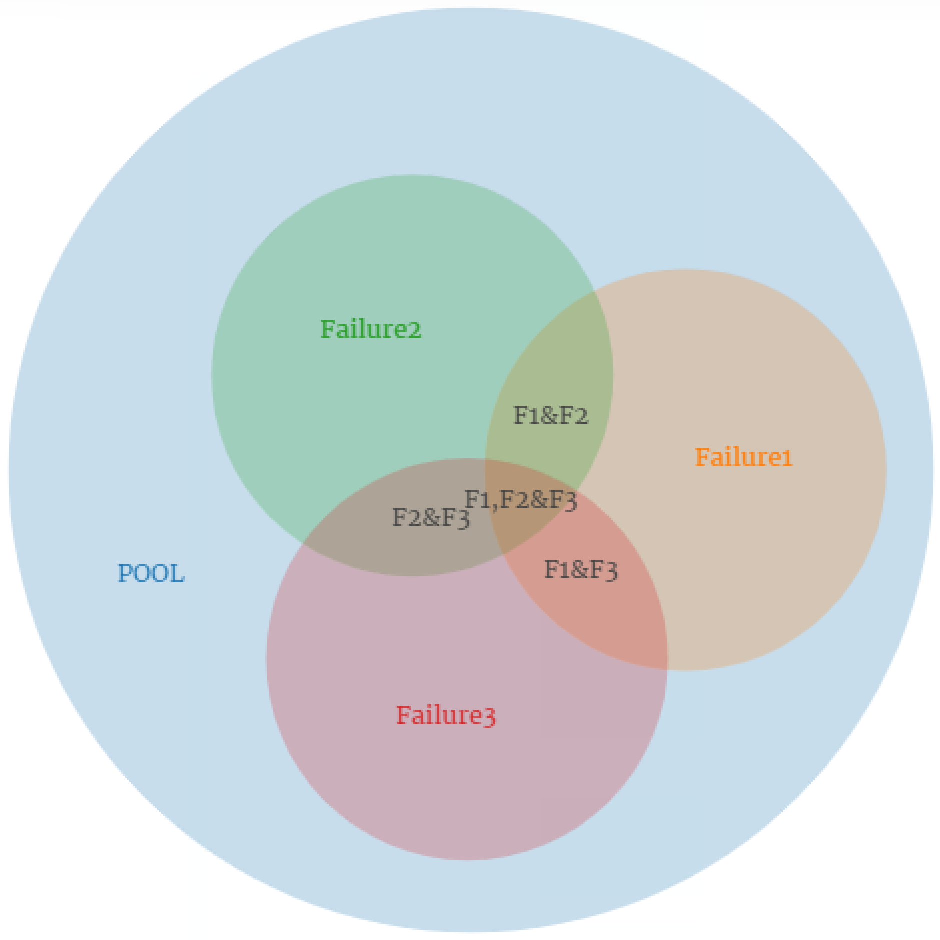


Figure 10: Venn Diagram of overlaps of 3 failures with 53-drive pool and EC size of 10.

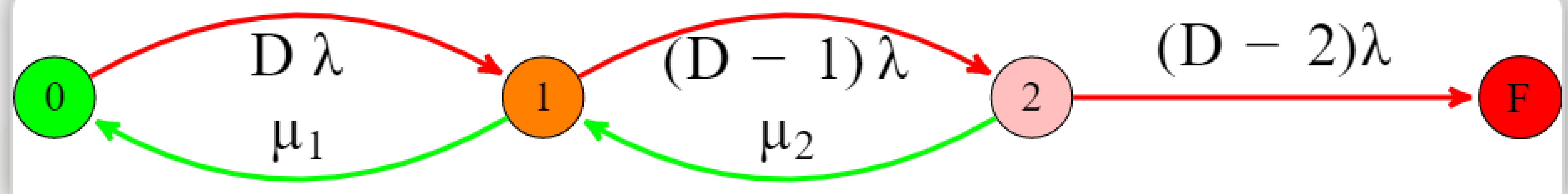


Figure 11: An approximate Markov chain for distributed parity of pool size D and redundancy 2. ... [+]

- Consider a system of pool-size D and EC size N, and redundancy c.
  - The overlaps are calculated for $D = 53$ and $N = 8 + 2$.
  - Geometric overlap areas scale with powers of N/D.
- Recovery rates: $\mu_1 \propto D$ and $\mu_2 \propto D^2$, failure rate: $\propto D$.
  - Increase in failure rate cancels with recovery speed up for $c = 1$.
  - Reli will benefit from ADAPT only if $c \geq 2$.
- ADAPT Reli can be expressed in terms of its RAID counterpart:
  - $\mathrm{MTTDL}_{\mathrm{dRAID}} = \left[\frac{D}{N}\right]^{\frac{c(c-1)}{2}} \mathrm{MTTDL}_{\mathrm{RAID}}$.
- For $D = 50$, $N = 10$, and $c = 2$: Adapt reli is 5x better than RAID6 reli.

Visualizing the damage

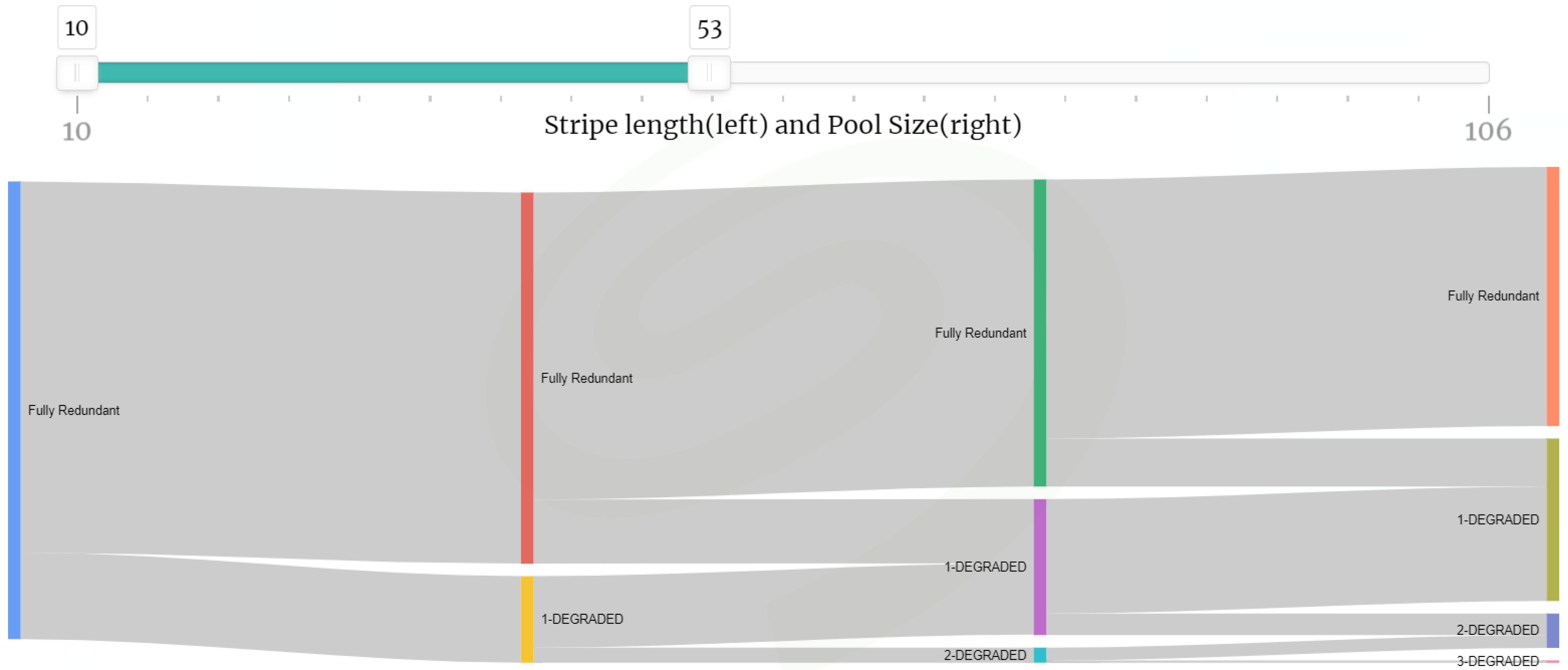


Figure 12: A Sankey Diagram of stripe degradation percentages with +2 Erasure coding. Hover on the image to see the numbers. ... [+]

Modeling Multi-Layer Erasure Coding

The overall data durability can be improved by implementing another layer of erasure coding. Top layer is composed of already erasure coded sub-elements.

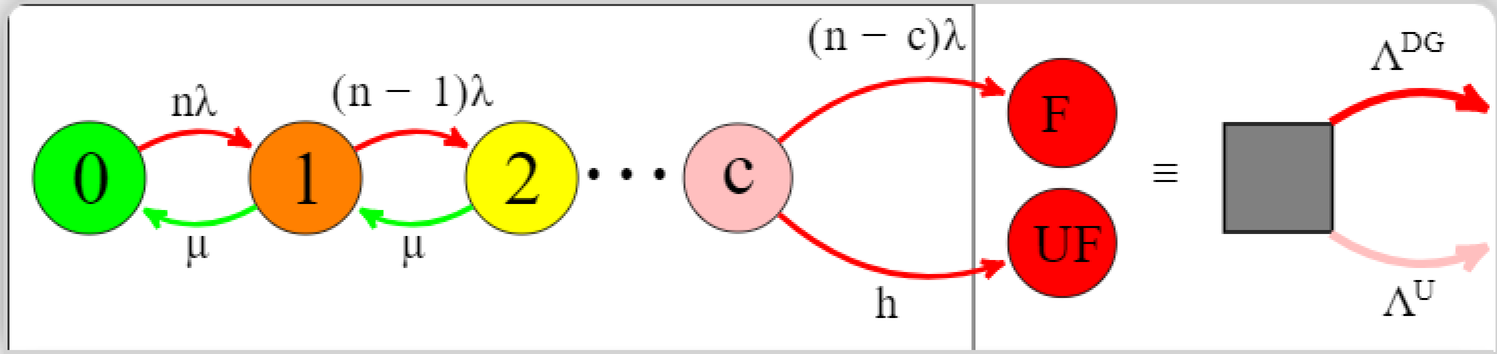


Figure 13: Left: the first layer of the overall erasure coding of size n and c reduncies.
Right: A block diagram representation of the erasure coded system as a single element. ... [+]

Figure 14: Calm on the surface, but always paddling like hell underneath. A lot of data paddling inside the enclosure, but from outside, it is just a very reliable petabyte drive.

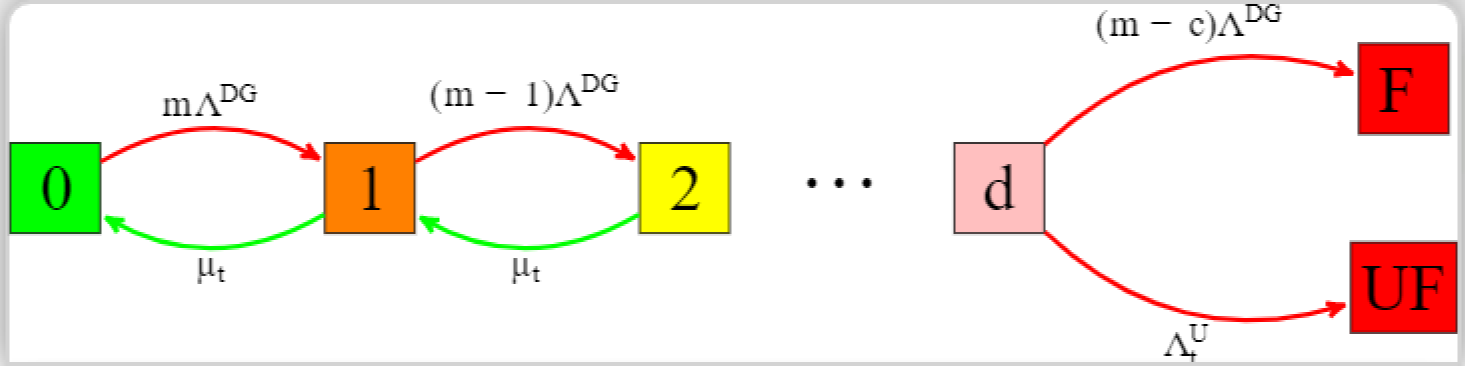


Figure 15: Top layer EC of size m and d reduncies built with self-erasure coded elements.

$$\frac{1}{\mathrm{MTTDL}_d^t} = \frac{1}{\mathrm{MTTDL}_{d,\mathrm{DG}}^t} + \frac{h_{m-d}^t}{\mathrm{MTTDL}_{d-1,\mathrm{DG}}^t}$$

$$\mathrm{MTTDL}_{d,\mathrm{DG}}^t = \left(\frac{\mu_t}{\Lambda^{\mathrm{DG}}}\right)^d \frac{(m-d-1)!}{\Lambda^{\mathrm{DG}} m!},$$

$h_{m-d}^t = 1 - (1 - \mathrm{UER})^{(n-c)(m-d)C}$ probability of URE(s).

- 16+2+Adapt (53-drive-pool) & 7+1 CORTX gives ~13 nines at the overall 73% capacity efficiency(27% overhead).
- ~12 nines can be reached with an 8+5 single layer EC at 62% capacity efficiency (38% overhead).

System Availability

Availability with no redundancy

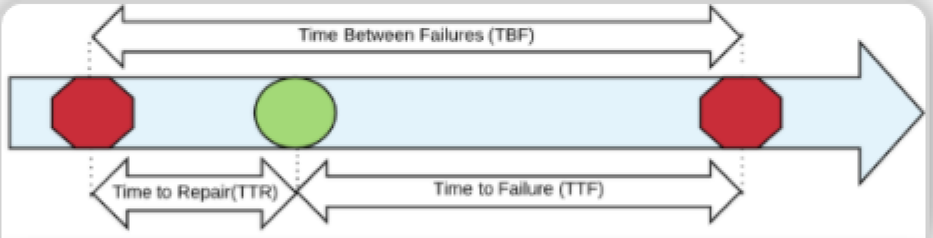


Figure 16: An illustration of the time scales.

- A set of n devices: $\overline{\text{MTTF}} = 1/(n\lambda_s)$, and $\overline{\text{MTTR}} = 1/\mu_s$.
- Fraction of time spent for repair: $\frac{\text{MTTR}}{\text{MTTF}+\text{MTTR}} \simeq \frac{\text{MTTR}}{\text{MTTF}} = \frac{n\lambda_s}{\mu_s}$.
- Availability=1-Fraction of time spent for repair: $$A_0 \simeq 1 - \frac{n\lambda_s}{\mu_s}. \quad (17)$$

Show a rigorous proof

Availability with one redundancy

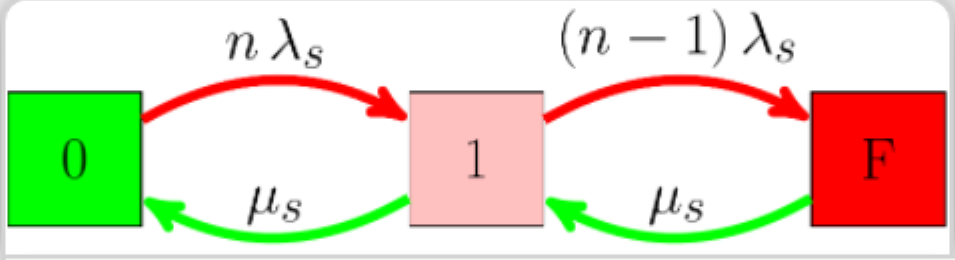


Figure 18: Markov Chain with one redundancy.

- With one redundancy, the data will be unavailable if 2^+ systems are down.
- Such a system can be modeled with the Markov chain shown on the left.
- The availability can be computed as $$A_1 = 1 - \left[n\frac{\lambda_s}{\mu_s}\right]\left[(n-1)\frac{\lambda_s}{\mu_s}\right] \quad (22)$$

Show a rigorous proof

Availability with c redundancies

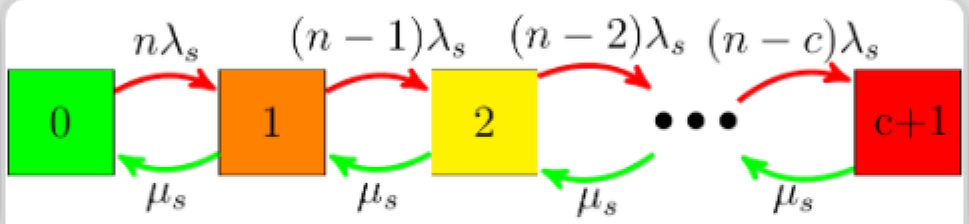


Figure 19: Markov Chain with c redundancies.

Recognizing the patterns in Eqs. (17) and (22) we can generalize the formula to a system with c redundancies:

$$A_c \simeq 1 - \frac{n!}{(n-c-1)!}\left[\frac{\lambda_s}{\mu_s}\right]^{c+1} \quad (28)$$

Appendix

Here we look at the data durability with dual actuator.

Press down arrow to navigate this sections.

Durability with Mach2

Show the GIF

- The reliability of RAID critically depends on the speed of the recovery.
- MACH2 doubles the data transfer $\Longrightarrow$ **2x/4x** reliability improvement for RAID5/6.

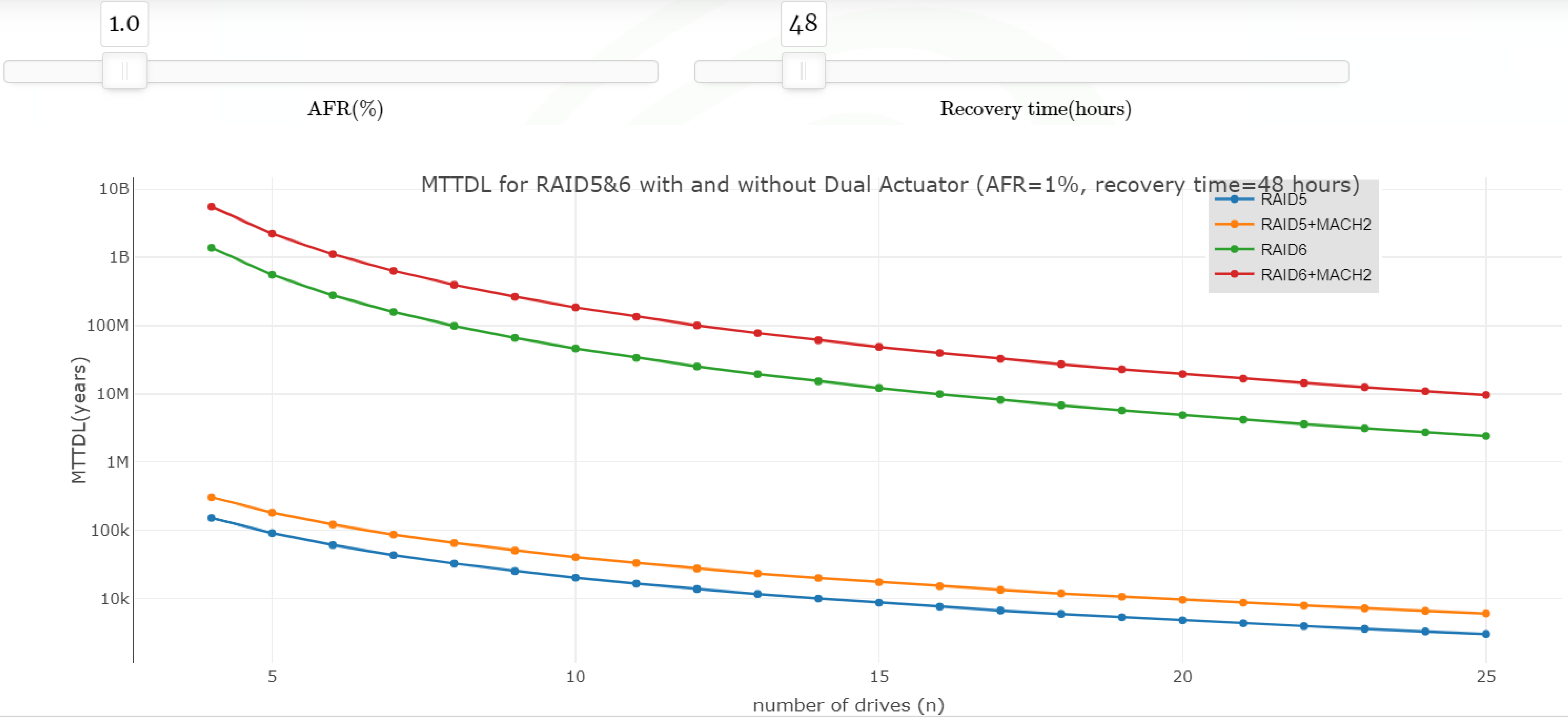


Figure 21: An interactive plot showing the gains with MACH2. ... [+]

Durability with ReMAN

- Consider with the simplest case[14]: one redundancy. Given a head/drive failure, there are two paths to losing data:
  - A second failure while recovering from a head failure: less likely due to faster recovery.
  - A second failure while recovering from a drive failure: more likely due to longer recovery.

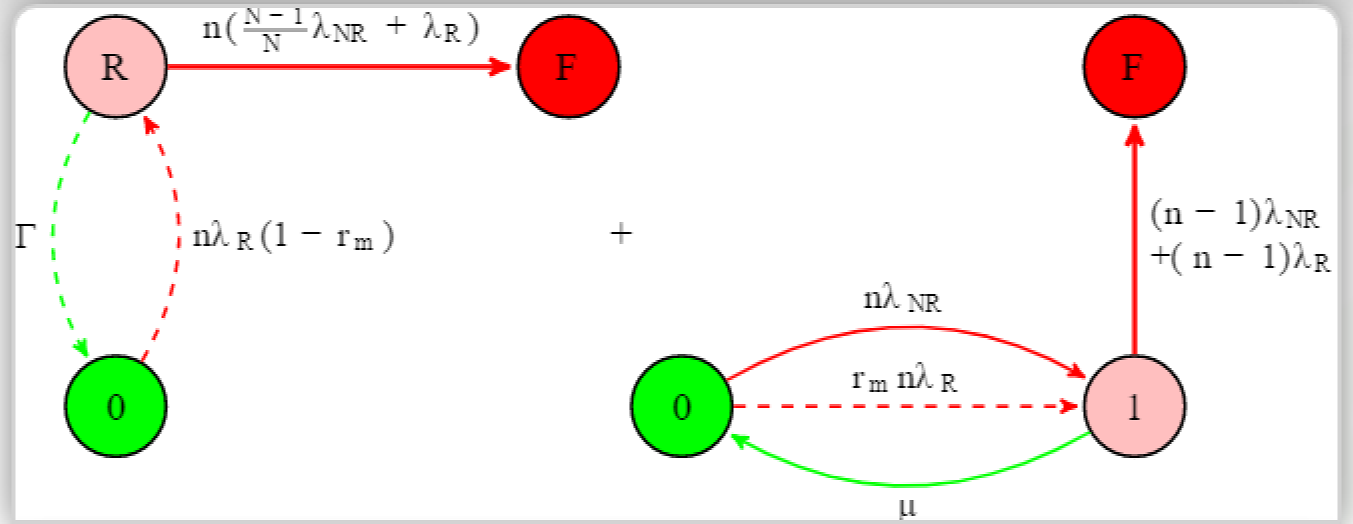


Figure 22: Markov chain split into two parallel paths: recovery from subcomponent failures and recovery from device failures.

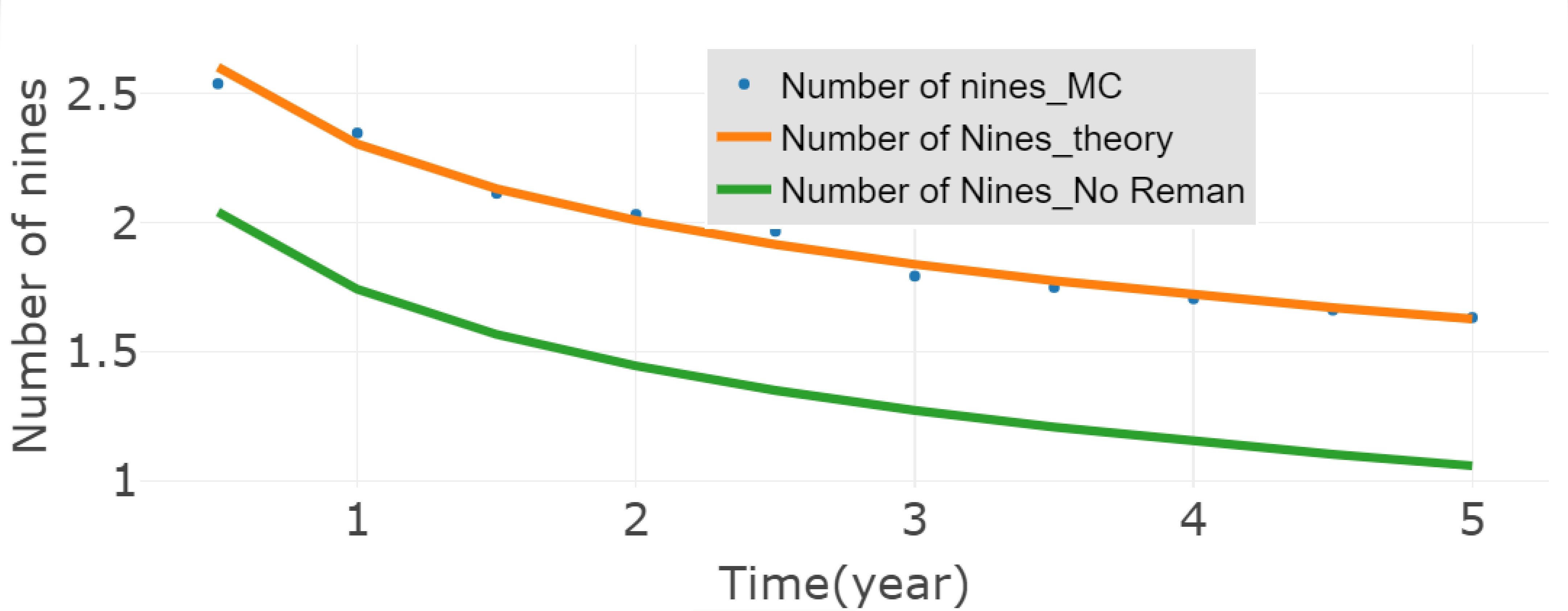
- The plot on the left shows two parallel Markov chains:
  - Left-most shows data recovery for a ReMan'able failure.
  - Right-most shows the standard recovery when a drive fails.
- r_m term represents the maximally ReMan'ed drive population.
  - ReMan'able failures on maximal drives trigger replacement.
  - This is represented by the dashed red curves.
- The transition rates are time dependent, and involve r_m functions.

- $\mathcal{R}_1^{(1)}(t) = exp\left(-\frac{\mathcal{C}}{2\Gamma}\left[t + \frac{1-e^{-2\lambda_R t}}{2\lambda_R}\right] - \frac{\mathcal{C}}{2\mu}\left[(1+2\kappa)t - \frac{1-e^{-2\lambda_R t}}{2\lambda_R}\right]\right)$ where $\mathcal{C} \equiv n^2\lambda_R^2(1+\kappa)$, and $\kappa \equiv \frac{\lambda_{NR}}{\lambda_R}$
  - n : number of drives in the EC scheme, N : number of heads per drive,
  - Γ : ReMan Repair rate, μ : drive repair rate, λ_R: ReMan'able failure rate, λ_{NR}: non-ReMan'able failure rate.
- This is the expression when we allow for 1 ReMan/drive. Similar formulas are calculated for $R_{max} = 2, 3$.

- We have an analytical model of Erasure Coded systems that support ReMan.
- The closed mathematical form can be computed instantly enabling a real-time web application.

Simulation with online ReMan

- Below is a comparison of Monte Carlo Simulation and theoretical results:
  - ReMan'able AFR= 8% and non ReMan'able AFR=2%.
  - Note that AFRs are take unrealistically high to show the functional behavior.
  - Assuming 1-ReMan per drive is allowed.
- The plot shows the theoretical prediction is remarkably accurate.
  - The gain in durability coming from ReMan is about 4x.



Number of nines with $\lambda_R = 1/12$ and $\lambda_{NR} = 1/50$ (1/year), which correspond to 8% and 2% AFR respectively.

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